

# Extra dimensions in particle physics

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Received: 8 January 2004 / Accepted: 8 February 2004 /  
Published Online: 25 February 2004 – © Springer-Verlag / Società Italiana di Fisica 2004

**Abstract.** Current problems in particle physics are reviewed from the viewpoint of theories possessing extra spatial dimensions.

## 1 Introduction

Today extra dimensions (ED) represent more a general framework where several aspects of particle physics have been reconsidered, rather than a unique and specific proposal for a coherent description of the fundamental interactions. The first motivation for the appearance of ED, namely the quest for unification of gravity and the other interactions [1], is still valid today. If we strictly adhere to this project, then at present the only viable candidate for a unified description of all interactions is string theory, which naturally requires ED. The idea of ED that we have in mind today has been deeply influenced by the developments in string theory: compactifications leading to a chiral fermion spectrum, localization of gauge and matter degrees of freedom on subspaces of the ED, relation between the topological properties of the compact space and the number of fermion families, localization of states around special points of the compact space and hierarchical Yukawa couplings, just to mention few examples. Remarkable theoretical progresses have also been obtained by developing models in field theory. For instance, in this context very fruitful tools for supersymmetry (SUSY) and gauge symmetry breaking have been developed, such as the Scherk-Schwarz [2] and the Hosotani [3] mechanisms. Also the physical properties of compactifications with non-factorizable space-time metric have been neatly worked out [4]. The conceptual and mathematical richness offered by these developments makes it possible to reconsider several specific problems that have not received a satisfactory answer in the four-dimensional context:

- **Hierarchy problem:** extreme weakness of gravity in comparison to the other interactions; gap between the electroweak scale and the Planck scale  $M_{Pl}$ .
- **Little hierarchy problem:** possible gap between the Higgs mass and the electroweak symmetry breaking scale.
- **Problems of conventional Grand Unified Theories (GUTs):** doublet-triplet splitting problem, proton lifetime, mass relations.

- **Flavour problem:** the architecture underlying the observed hierarchy of fermion masses and mixing angles.
- **Cosmological constant problem:** small curvature of the observed space-time and its relation to the dynamics of particle interactions.

In this talk I will review the viewpoint on these problems offered by theories with ED, stressing the most recent developments in the field.

## 2 Hierarchy problem

### 2.1 Large extra dimensions

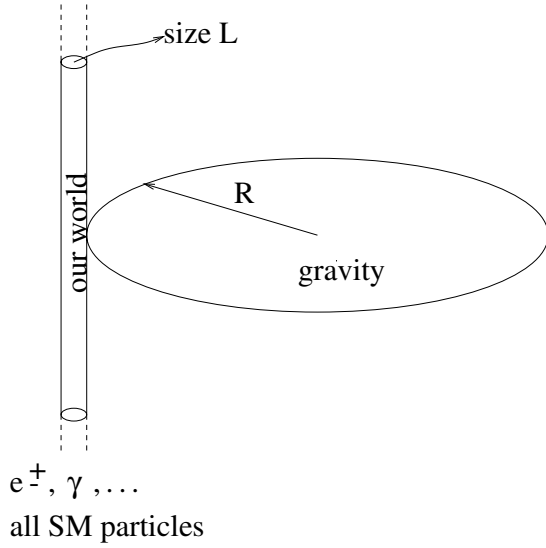
The hierarchy problem can be reformulated in the context of large extra dimensions (LED) [5]. In the LED scenario there is only one fundamental energy scale for particle interactions: the TeV scale. Gravity describes the geometry of a  $D = 4 + \delta$  dimensional space-time where  $\delta$  dimensions are compactified, for instance, on an isotropic torus  $T^\delta$  of radius  $R$ . The D-dimensional Planck mass  $M_D$  is of the order 1 TeV. All the other degrees of freedom of the standard model (SM) are assumed to live on a four-dimensional subspace, usually called brane, of the full D-dimensional space-time (see Fig. 1). The gravitational potential  $V(r)$  between two massive particles at a distance  $r$  has two regimes. For  $r \ll R$ , the lines of the gravitational field extend isotropically in all directions and

$$V(r) \propto \frac{1}{M_D^{2+\delta}} \frac{1}{r^{1+\delta}} \quad , \quad (1)$$

whereas for  $r \gg R$ , the lines are squeezed along the usual four dimensions:

$$V(r) \propto \frac{1}{M_D^{2+\delta} V_\delta} \frac{1}{r} \quad , \quad (2)$$

where  $V_\delta$  is the volume of the compact space. As a consequence the four-dimensional Planck mass  $M_{Pl}$  is given by



**Fig. 1.** Pictorial view of the generic set-up considered in this review. Gravity has access to the full space-time characterized by extra spatial dimensions of typical size  $R$ . SM particles are localized in subspaces of the full space-time, whose typical extension in the extra space is  $L \ll R$ . When discussing LED, we will set  $L = 0$

the relation<sup>1</sup>:

$$\left(\frac{M_{Pl}}{M_D}\right)^2 = M_D^\delta V_\delta \quad . \quad (3)$$

Therefore a huge hierarchy between  $M_{Pl}$  and  $M_D \approx 1$  TeV is possible if the volume  $V_\delta$  of the extra space is much larger than naively expected,  $M_D^{-\delta}$ . In units where  $M_D = 1$  we have

$$\frac{1}{M_{Pl}} = \frac{1}{\sqrt{V_\delta}} \quad , \quad (4)$$

indicating that four-dimensional gravity is weak because the four-dimensional graviton wave function is diluted in a big extra space. The  $1/\sqrt{V_\delta}$  suppression of the gravitational constant can also be understood as the normalization factor of the wave function for the graviton zero mode. Solving the hierarchy problem now requires explaining why  $V_\delta \gg M_D^{-\delta}$ . This is a dynamical problem, since in higher-dimensional theories of gravity the volume  $V_\delta$  is the vacuum expectation value (VEV) of a field. For instance in a five-dimensional theory, the compactification radius  $R$  is determined by the VEV of the radion field  $r(x)$ , the (zero-mode of the) 55 component of the full space-time metric:

$$\langle r(x) \rangle = RM_D \quad , \quad (5)$$

in close analogy to the Fermi scale, determined in the electroweak theory by the VEV of the Higgs field. In a realistic theory of gravity like string theory, the solution to this

<sup>1</sup> Several conventions exist in the literature to introduce the fundamental scale of gravity  $M_D$ . In this review the relation (3) defines the (reduced) D-dimensional Planck mass  $M_D$  in terms of the reduced four-dimensional Planck mass  $M_{Pl} = 2.4 \times 10^{18}$  GeV.

**Table 1.** Compactification radius  $R$  and compactification scale  $R^{-1}$  as a function of the number  $\delta$  of ED, for  $M_D = 1$  TeV.

$\delta$	$R$	$R^{-1}$
1	$10^8$ Km	$10^{-18}$ eV
2	0.1 mm	$10^{-3}$ eV
3	$10^{-6}$ mm	100 eV
...	...	...
7	$10^{-12}$ mm	100 MeV

dynamical problem requires finding the minimum of an energy functional depending simultaneously upon many moduli, a rather formidable task.

In Table 1, we read the radius  $R$  and the compactification scale  $R^{-1}$  as a function of the number  $\delta$  of ED, assuming  $M_D = 1$  TeV, an isotropic toroidal compactification and a flat background metric. Of special interest is the case  $\delta = 2$ , which predicts deviations from the Newton’s law at distances around 1 mm. For  $\delta > 2$  such deviations would occur at much smaller length scales, outside the range of the present experimental possibilities. For the values of  $\delta$  quoted in Table 1, the compactification scale is quite small and the Kaluza-Klein (KK) modes of gravitons can be produced both at colliders and in processes of astrophysical and/or cosmological relevance.

### 2.1.1 Deviations from Newton’s law

Modifications of the laws of gravity at small distances are currently under intense investigation. In experimental searches, deviations from the Newton’s law are parametrized by the modified gravitational potential

$$V(r) = -\frac{G_N m}{r} \left( 1 + \alpha e^{-\frac{r}{\lambda}} \right) \quad , \quad (6)$$

( $G_N = 1/(8\pi M_{Pl}^2)$  is the Newton constant) in terms of the relative strength  $\alpha$  and the range  $\lambda$  of the additional contribution. Precisely this kind of deviation is predicted by LED for  $r \geq \lambda$ . The range coincides with the wavelength of the first KK graviton mode,  $\lambda = 2\pi R$ , while the relative strength is given by the degeneracy of the first KK

level:  $\alpha = 2\delta$ . At present the best sensitivity in the range  $\lambda \approx 100 \mu\text{m}$  has been attained by the torsion pendulum realized by the Eöt-Wash group. For  $\delta = 2$  they obtained the limit [6]

$$\lambda < 150 \mu\text{m} \quad (95\% \text{ CL}) \quad , \quad (7)$$

already in the range where deviations are expected if the  $M_D$  is close to 1 TeV. Future improvements represent a real challenge from the experimental viewpoint, but their impact could be extremely important, since deviations around the currently explored range are also expected in other theoretically motivated scenarios, as we shall see later on. The next planned experiments aim to reach a sensitivity on  $\lambda$  in the range (30-50)  $\mu\text{m}$  for  $\alpha = 4$  [7], thus probing  $M_D$  up to  $3 \div 4$  TeV.

### 2.1.2 Neutrino masses

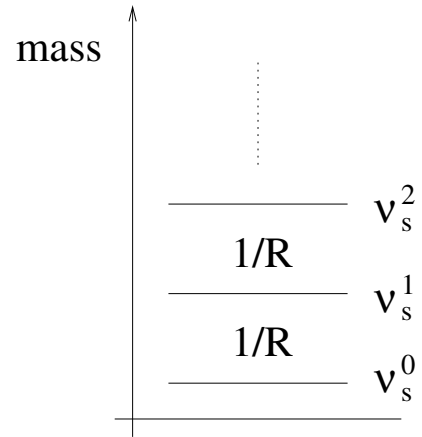
LED provide a nice explanation of the smallness of neutrino masses [8]. If right-handed neutrinos  $\nu_s$  ( $s \equiv \text{singlet}$  under the SM gauge interactions) exist, at variance with the charged fermion fields, they are allowed to live in the bulk of a LED. In this case, just as for the graviton, their four-dimensional modes in the Fourier expansion carry a suppression factor  $1/\sqrt{V_\delta}$ :

$$\nu_s(x, y) = \frac{\nu_s^{(0)}(x)}{\sqrt{V_\delta}} + \dots \quad (8)$$

By taking into account the relation (3), the Dirac neutrino masses originating from the Yukawa coupling with the Higgs doublet are given by:

$$\mathcal{L}_{Yuk} = \frac{y_\nu v}{\sqrt{2}} \left( \frac{M_D}{M_{Pl}} \right) \nu_a(x) \nu_s^{(0)}(x) + \dots \quad (9)$$

where  $\nu_a(x)$  are the active four-dimensional neutrinos. The resulting Dirac neutrino masses are much smaller than the charged fermion masses and the observed smallness of neutrino masses is explained, if there are no additional contributions. The latter might arise from dimension five operators associated to the violation of the lepton number. In the absence of a sufficiently large fundamental scale, new mechanisms should be introduced to guarantee the desired suppression of these operators [9, 10]. Testing this idea and detecting the higher-dimensional origin of neutrino masses would represent a clean signature of the LED scenario. Experimentally, this is only possible if some  $\nu_s^{(n)}$  (see Fig. 2) is sufficiently light,  $1/R \leq \sqrt{\Delta m_{atm, sol}^2} \approx 0.01$  eV, which is not unconceivable if there is one dominantly large ED with  $R \approx 0.02$  mm. In such a case few  $\nu_s^{(n)}$  levels may take part in neutrino oscillations and produce observable effects. Unfortunately present data disfavour this exciting possibility. First of all, there are clear indications in favour of oscillations among active neutrinos, both in the solar and in the atmospheric sectors [11]. Moreover SN1987A excludes the large mixing angle between active and sterile neutrinos that would



**Fig. 2.** Mass spectrum of right-handed neutrinos  $\nu_s^{(n)}$

be needed to reproduce, for instance, the solar data in this scenario [12]. Therefore the effects of  $\nu_s^{(n)}$  on neutrino oscillations are sub-dominant, if present at all. Indeed, if the KK levels  $\nu_s^{(n)}$  are much heavier than the mass scale relevant for neutrino oscillations, they decouple from the low-energy theory and the higher-dimensional origin of neutrino masses becomes undetectable.

### 2.1.3 Signals at colliders

Experimental signatures of LED at present and future colliders are well understood by now [13] and an intense experimental search is currently under way. The existence of light KK graviton modes leads to two kinds of effects. The first one is the direct production of KK gravitons in association with a photon or a jet, giving rise to a signal characterized by missing energy (or transverse energy) plus a single photon or a jet. The cross section for the production of a single KK mode is depleted by the four-dimensional gravitational coupling,  $1/M_{Pl}^2$ , but the lightness of each individual graviton mode makes it possible to sum over a large number of indistinguishable final states and the cross section for the expected signal scales as:

$$\sigma \approx \frac{E^\delta}{M_D^{\delta+2}} \quad (10)$$

in terms of the available center of mass energy  $E$ . The present lower bound on  $M_D$ , listed in Table 2, are dominated by the searches for  $e^+e^- \rightarrow \gamma + \cancel{E}$  at LEP and  $p\bar{p} \rightarrow \gamma + \cancel{E}_T$  at Tevatron. For  $\delta = 2, 3, 4$  the limits from LEP are slightly better than those from Tevatron, the opposite occurring for  $\delta = 5, 6$ . Recently, comparable limits have also been obtained at Tevatron [14], by looking for final states with missing transverse energy and one or two high-energy jets.

This signal is theoretically clean. Moreover, as long as the energy  $E$  does not exceed  $M_D$ , thus remaining within the domain of validity of the low-energy effective theory, the predictions do not depend on the ultraviolet properties of the theory.

**Table 2.** Combined 95% CL limits on  $M_D$  from LEP and Tevatron data, from [15]. Notice that  $M_D$  here is related to  $M_D$  [15] by  $M_D$  [15] =  $(2\pi)^{\delta/(2+\delta)} M_D$

$\delta$	2	3	4	5	6
$M_D$ (TeV)	0.57	0.36	0.26	0.19	0.16

A second type of effect is that induced by virtual KK graviton exchange. At variance with the previous one, such effect is sensitive to the ultraviolet physics: the amplitudes diverge already at the tree level for  $\delta \geq 2$ . In string theory the tree-level amplitudes from graviton exchange correspond to one-loop amplitudes for the exchange of open string excitations. They are compactification dependent and also sub-leading compared to open string tree-level exchanges [16]. In a more model-independent approach, based on a low-energy effective field theory, the best that can be done is to parametrize these effects in terms of effective higher-dimensional operators. If we focus on the sector of the theory including only fermions and gauge bosons, there are only two independent operators up to dimension  $d = 8$  [17, 15]:

$$\frac{c_\tau}{2} \left( T_{\mu\nu} T^{\mu\nu} - \frac{T_\mu^\mu T_\nu^\nu}{2+\delta} \right) \quad (d=8) \quad , \quad (11)$$

$$\frac{c_Y}{2} \left( \sum_f \bar{f} \gamma^\mu \gamma_5 f \right)^2 \quad (d=6) \quad , \quad (12)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor and the sum over  $f$  extends to all fermions. The dimension eight operator arises from graviton exchange at the tree-level. It contributes to dilepton production at LEP, to diphoton production at Tevatron and to the scattering  $e^\pm p \rightarrow e^\pm p$  at Hera. The present data imply the bound [15]:

$$\left( \frac{8}{|c_\tau|} \right)^{\frac{1}{4}} > 1.3 \text{ TeV} \quad . \quad (13)$$

The dimension six operator arises at one-loop. It is even under charge conjugation and singlet under all gauge and global symmetries. It is bounded mainly from the search of contact interactions at LEP, dijet and Drell-Yan production at Tevatron and  $e^\pm p \rightarrow e^\pm p$  at Hera [15]:

$$\left( \frac{4\pi}{|c_Y|} \right)^{\frac{1}{2}} > (16 \div 21) \text{ TeV} \quad , \quad (14)$$

where the two quoted bounds refer, respectively, to a negative and a positive  $c_Y$ . In terms of the fundamental scale  $M_D$  and the cut-off  $\Lambda$  of the effective theory,  $c_\tau$  and  $c_Y$  are expected to scale as:

$$c_\tau \approx \frac{\Lambda^{\delta-2}}{M_D^{\delta+2}} \quad c_Y \approx \frac{\Lambda^{2\delta+2}}{M_D^{2\delta+4}} \quad , \quad (15)$$

(more refined estimates can be found in [15]). If we naively set  $\Lambda \approx M_D$ , then the present limit on  $c_Y$  from 14 would provide the strongest collider bound on  $M_D$ . In a conservative analysis  $M_D$  and  $\Lambda$  should be kept as independent. In this case, assuming  $M_D \approx 1$  TeV, we see that the bound (14) and the estimate (15) imply that  $\Lambda$  should be considerably smaller than  $M_D$ . Such a situation, where the cut-off scale is required to be much smaller than the mass scale characterizing the low-energy effective theory, is not uncommon in particle physics. For instance the naive estimate of the amplitude for  $K_L^0 \rightarrow \mu^+ \mu^-$  in the Fermi theory gives  $G_F^2 \Lambda^2$  and data require  $\Lambda \ll 1/\sqrt{G_F}$ . Indeed here the role of  $\Lambda$  is played by the charm mass, as a consequence of the GIM mechanism. Similarly, the stringent bound on  $c_Y$  could indicate that the modes needed to cure the ultraviolet behaviour of amplitudes with KK graviton exchange are possibly quite light, if the fundamental scale  $M_D$  is close to the TeV range.

The present sensitivity will be considerably extended by future colliders, like LHC, that could probe  $M_D$  up to about 3.4(2.3) TeV, for  $\delta = 2(4)$  [13]. The most promising channel is single jet plus missing transverse energy. Estimates based on the low-energy effective theory become questionable for  $\delta \geq 5$ . If the energy at future colliders became comparable to the fundamental scale of gravity  $M_D$ , production and decay of black holes could take place in our laboratories, with expectations that have been reviewed by Landsberg at this conference [18]. In an even more remote future, collisions at trasplanckian energies could provide a robust check of these ideas, especially for the possibility of dealing with gravity effects in a regime dominated by the classical approximation [19].

#### 2.1.4 Limits from astrophysics

Today the most severe limits on  $M_D$  come from astrophysics and in particular from processes that can influence supernova formation and the evolution of the daughter neutron star. There are three relevant processes. The first one is KK graviton production during the explosion of a supernova, whose typical temperature of approximately 50 MeV makes kinematically accessible the KK levels for the compactification scales listed in Table 1.

The amount of energy carried away by KK gravitons cannot deplete too much that associated to neutrinos, whose flux was observed in SN1987A. A competing process is KK graviton production, mainly induced by nucleon-nucleon bremsstrahlung  $NN \rightarrow NN \text{ graviton}$ , and controlled in the low-energy approximation by  $M_D$ . This process should be adequately suppressed. If KK graviton production during supernova explosions takes place, then a large fraction of gravitons remains trapped in the neutron star halo and the subsequent decay of gravitons into photons produces a diffuse  $\gamma$  radiation, which is bounded by the existing measurements by the EGRET satellite. The photon flux should also not overheat the neutron star surface. The corresponding limit on  $M_D$  depends on the assumed decay properties of the massive gravitons. These limits are summarized, for  $\delta = 2, 3, 4$  in Table 3. These

**Table 3.** Lower bound on  $M_D$ , in TeV, from astrophysical processes [20]

	$\delta = 2$	$\delta = 3$	$\delta = 4$
SN cooling (SN1987A)	8.9	0.66	0.01
Diffuse gamma rays	38.6	2.65	0.43
NS heat excess	701	25.5	2.77

bounds rapidly softens for higher  $\delta$ , due to the energy dependence of the relevant cross-sections in the low-energy approximation. They are strictly related to the spectrum of KK gravitons, which, as we can see from Table 1, has no sizeable gap compared to the typical energy of the astrophysical processes considered here.

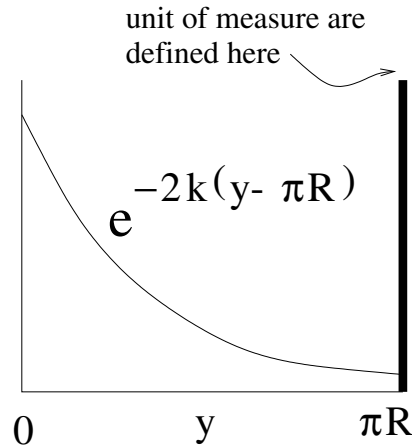
We also recall that KK graviton production can largely affect the universe evolution [21]. Going backwards in time, for  $M_D \approx 1$  TeV, the universe has a standard evolution only up to a temperature  $T_*$  given approximately by 10 MeV, if  $\delta = 2$  and by 10 GeV, if  $\delta = 3$ . For higher temperatures the production of KK gravitons replaces the adiabatic expansion as the main source of cooling. While such low temperatures are still consistent with the big-bang nucleosynthesis, they render both inflation and baryogenesis difficult to implement.

In summary,  $M_D \approx 1$  TeV still represents a viable possibility if  $\delta \geq 4$ , while for lower  $\delta$  and in particular for the special case  $\delta = 2$ , it is difficult to reconcile the expectations of LED, at least in the simplest version discussed here, with astrophysical data.

## 2.2 Warped compactification

Both astrophysical and cosmological problems are evaded if the spectrum of KK gravitons has a sufficient gap. For instance a gap higher than the temperature of the hottest present astrophysical object, approximately 100 MeV, removes all the astrophysical bounds discussed above. Such a gap can be obtained in several ways. Up to now we have assumed an isotropic toroidal compactification. In general, the KK spectrum depends not only on the overall volume  $V_\delta$ , but also on the moduli that define the shape of the compact space. For particular choices of these moduli, the KK spectrum displays the desired gap [22].

An additional assumption that has been exploited up to this point is that of a factorized metric for the space-time. This assumption is no longer justified if the underlying geometry admits walls (also referred to as branes) carrying some energy density. Then, by the laws of gen-



**Fig. 3.** Dependence of the warping factor on the extra coordinate  $y$ . With the adopted parametrization the warping factor is equal to one at the brane in  $y = \pi R$ . Mass parameters are measured in units used by the observer at  $y = \pi R$

eral relativity, the background metric is warped and the relation between  $M_{Pl}$  and  $M_D$  is modified [4]. For instance, in the Randall-Sundrum set-up, the metric can be parametrized as:

$$ds^2 = e^{2k(y - \pi R)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad , \quad (16)$$

where  $\eta_{\mu\nu}$  is the four-dimensional Minkowski metric and  $k^{-1}$  is the radius of the AdS space. Notice that we have rescaled the coordinates  $x^\mu$  by the overall factor  $e^{k\pi R}$ , compared to the most popular parametrization of the Randall-Sundrum metric. As a result, all mass parameters are now measured in units of the typical mass scale at  $y = \pi R$ , the TeV (see Fig. 3). The masses  $M_{Pl}$  and  $M_D$  are now related by:

$$\left( \frac{M_{Pl}}{M_D} \right)^2 = M_D \frac{(e^{2k\pi R} - 1)}{k} \quad . \quad (17)$$

This relation can be viewed, for  $\delta = 1$  as a generalization of the one given in (3) for a factorizable metric. Indeed, by sending  $k$  to zero the metric (16) becomes flat and we recover (3). By comparing (17) and (3) we see that the factor multiplying  $M_D$  on the right hand side of (17) plays, in a loose sense, the role of the volume  $V_\delta$  of the compact space, measured in units used by the observer at  $y = \pi R$ . In this case the dependence of the “volume” on the radius  $R$  is exponential<sup>2</sup>. Remarkably, we do not need a huge radius  $R$  to achieve a large hierarchy between  $M_{Pl}$  and  $M_D \approx 1$  TeV. If  $k$  and  $M_D$  are comparable, then  $R \approx 10k^{-1}$  does the job. The KK graviton levels, controlled by  $1/R$  in the present parametrization, start now naturally at the TeV scale. Astrophysical and cosmological bounds do not apply. Signals at colliders are quite different from those discussed in the LED case. The KK gravitons have couplings suppressed by the TeV scale, not by  $M_{Pl}$ . Their

<sup>2</sup> Such a dependence is also found in other geometries of the internal space [23].

levels are not uniformly spaced and they are expected to produce resonance enhancements in Drell-Yan processes. A portion of the parameter space has already been probed at the Tevatron collider by CDF, by searching for heavy graviton decays into dilepton and dijet final states [24]. In this model the radion mass is expected to be in the range  $0.1 - 1$  TeV, the exact value depending on the specific mechanism that stabilizes the radius  $R$ . It can be produced by gluon fusion and it mainly decays into a dijet or a  $ZZ$  pair, as the Higgs boson. No significant bounds can be extracted from the present radion search.

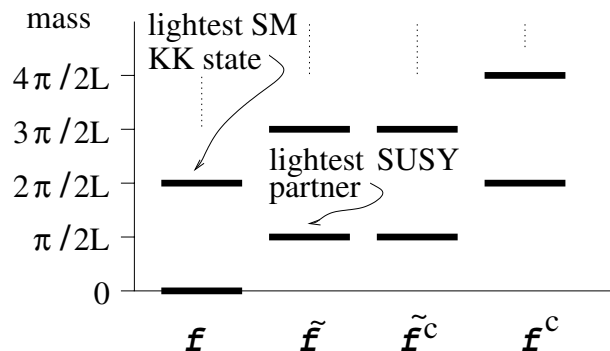
The Randall-Sundrum setup provides an interesting alternative to LED. It avoids the tuning of geometrical parameters versus  $M_D$  that is still needed in LED to reproduce the hierarchy between  $M_{Pl}$  and the electroweak scale. It overcomes the difficulties related to the presence in LED of a “continuum” of KK graviton states.

### 3 Little hierarchy problem

The assumption that the SM fields are confined on a four-dimensional brane that does not extend into the extra space is too restrictive. Strong and electroweak interactions have been successfully tested only up to energies of order TeV. Therefore a part of or all the SM fields might have access to extra dimensions of typical size  $L \leq (\text{TeV})^{-1}$  (see Fig. 1). There are several theoretical motivations for extra dimensions at the TeV scale. Already long ago, it was observed that one way to achieve supersymmetry breaking with sparticle masses in the TeV range, is through a suitable TeV compactification [25]. Here I will discuss a more recent development, related to the so-called ‘little’ hierarchy problem. On the one hand, the present data provide an indirect evidence for a gap between the Higgs mass  $m_H$ , required to be small by the precision tests, and the electroweak breaking scale, required to be relatively large by the unsuccessful direct search for new physics. On the other hand, from the solution of the ‘big’ hierarchy problem, we would naively expect that a light Higgs boson should require new light weakly interacting particles (e.g. the chargino in the MSSM), that have not been revealed so far. This gap is not so large and it can be filled either by a moderate fine-tuning of the parameters in the underlying theory, or by looking for specific theories where it can be naturally produced. Extra dimensions at the TeV scale provide in principle a framework for these more natural theories. Indeed, new weakly interacting states show up at the TeV scale, whereas the Higgs mass can be kept lighter by some symmetry.

#### 3.1 Higgs mass protected by SUSY

In the last years there has been a growing interest in five-dimensional models where supersymmetry is broken by boundary conditions on an interval of size  $L \approx 1 \text{ TeV}^{-1}$ . Supersymmetry breaking by boundary condition can reduce the arbitrariness in the soft breaking sector, thus making the model more predictable [26]. Moreover such

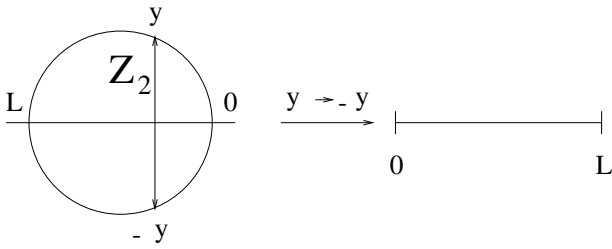


**Fig. 4.** Mass spectrum of the model in [30] for a generic matter multiplet:  $f$  and  $\tilde{f}$  are respectively fermion and scalar components of a chiral  $N = 1$  SUSY multiplet;  $f^c$  and  $\tilde{f}^c$  are additional components required by supersymmetry in  $D=5$

a possibility provides an alternative to the MSSM with universal boundary condition at the grand unified scale to study the interplay between supersymmetry and electroweak symmetry breaking [26,27]. Also, such a set-up demonstrates very useful to study important theoretical issues such as the problems of cancellation of quadratic divergences [28] and of gauge anomalies [29]. As in the MSSM, the electroweak symmetry breaking can be triggered by the top Yukawa coupling. For instance, in a particular model [30] belonging to this class, the Higgs mass is finite and calculable in terms of two parameters, the length  $L$  of the extra dimension and a mass parameter  $M$  that is responsible for the localization of the wave function for the zero mode of the top quark. By including not only leading terms, but also two-loop corrections originating from the top Yukawa coupling and the strong coupling constant, a Higgs mass in the relatively narrow range  $m_H = (110 \div 125) \text{ GeV}$  is found, for  $L^{-1} = (2 \div 4) \text{ TeV}$  and  $2 \leq LM \leq 4$ . The model is characterized by the spectrum of KK excitations displayed in Fig. 4. The KK tower of each ordinary fermion is accompanied by a tower for the associated SUSY partner. The two towers have the same spacing,  $\pi/L$ , but they are shifted by  $\pi/(2L)$ . Two additional towers are required by SUSY in five dimensions. The detection of this pattern would provide a distinctive experimental signature of the model. A peculiar feature of the model is that all the degrees of freedom are in the bulk (models of this type are said to have universal extra dimensions [31]), and the only localized interactions are the Yukawa ones. Therefore momentum along the fifth dimension is conserved by gauge interactions. No single KK mode can be produced through gauge interactions and no four-fermion operator arises from tree-level KK gauge boson exchange. This property softens the experimental bounds on universal extra dimensions.

#### 3.2 Higgs mass protected by gauge symmetry: Higgs-gauge unification

More than twenty years ago Manton [32] suggested that the Higgs field could be identified with the extra components of a gauge vector boson living in more than four



**Fig. 5.** A  $Z_2$  parity symmetry halves a circle  $S$  and all the degrees of freedom originally defined on  $S$

dimensions. In this case the same symmetry that protects gauge vector bosons from acquiring a mass, could help in preventing large quantum corrections to the Higgs mass [33, 34].

A simple example of how this idea can be practically implemented is a Yang-Mills theory in  $D > 4$  dimensions with gauge group  $SU(3)$ . The gauge vector bosons can be described by a  $3 \times 3$  hermitian matrix  $A_M$  transforming in the adjoint representation of  $SU(3)$ :

$$A_M = \begin{pmatrix} A_M^a & A_M^{\hat{a}} \\ A_M^{\hat{a}} & A_M^a \end{pmatrix}. \quad (18)$$

The vector bosons  $A_M^a$  ( $a = 1, 2, 3, 8$ ), lying along the diagonal  $2 \times 2$  and  $1 \times 1$  blocks, are related to the  $SU(2) \times U(1)$  diagonal subgroup of  $SU(3)$ , to be identified with the gauge group of the SM, while the fields  $A_M^{\hat{a}}$  ( $\hat{a} = 4, 5, 6, 7$ ), belonging to the off-diagonal blocks, are instead associated to the remaining generators of  $SU(3)$ . From the four-dimensional point of view,  $A_M$  describes both vector bosons ( $M[\equiv \mu] = 0, 1, 2, 3$ ) and scalars ( $M[\equiv m] = 5, \dots$ ). Out of all these degrees of freedom, in the low-energy theory we would like to keep  $A_\mu^a$ , that have the quantum numbers of the electroweak gauge vector bosons  $\gamma$ ,  $Z$  and  $W^\pm$ , and  $A_m^{\hat{a}}$ , transforming exactly as complex Higgs doublets  $H_m$ , under  $SU(2) \times U(1)$ . On the contrary, the particles related to  $A_\mu^{\hat{a}}$  and  $A_m^a$  are unseen and should be somehow eliminated from the low-energy description. Recalling that all these fields have a Fourier expansion containing a zero mode plus a KK tower, we would like to keep the zero modes in the desired sector and project away the zero modes for the unseen states. Such a projection becomes very natural [34, 35] if the compactified space is an orbifold  $S/Z_2$  (see Fig. 5), where all the fields are required to have a specific parity under the inversion of the fifth coordinate:  $y \rightarrow -y$ . It is sufficient to require that the fields describing the unwanted states are odd, and all the remaining ones are even. Such a parity assignment is compatible both with the five-dimensional  $SU(3)$  gauge symmetry and the five-dimensional Lorentz invariance. To the four-dimensional observer, having access only to the massless modes, the  $SU(3)$  symmetry appears to be broken down to  $SU(2) \times U(1)$ . Moreover she/he will count one Higgs doublet  $H$  for each ED.

In order to promote this fascinating interpretation of the gauge-Higgs system to a realistic theory, a number of problems should be overcome. First, in this example, the request that  $H$  has the correct hypercharge leads to  $\sin^2 \theta_W = 3/4$ , a rather bad starting point. A value of  $\sin^2 \theta_W(m_Z)$  closer to the experimental result can be obtained by replacing  $SU(3)$  with another group. For instance, the exceptional group  $G_2$  leads to  $\sin^2 \theta_W = 1/4$  [36]. No large logarithms are expected to modify the tree-level prediction of  $\sin^2 \theta_W$ , although some corrections can arise from brane contributions. Second, electroweak symmetry breaking requires a self-coupling in the scalar sector. In  $D = 5$  this coupling can be provided by D-terms, if the model is supersymmetric [37]. In  $D = 6$  the desired coupling is contained in the kinetic term for the higher dimensional gauge bosons. Third, crucial to the whole approach is the absence of quadratic divergences that could upset the Higgs lightness. A key feature of these models is a residual gauge symmetry associated to the broken generators [38]. It acts on the Higgs field as

$$A_5^{\hat{a}} \rightarrow A_5^{\hat{a}} + \partial_5 \alpha^{\hat{a}} \quad (19)$$

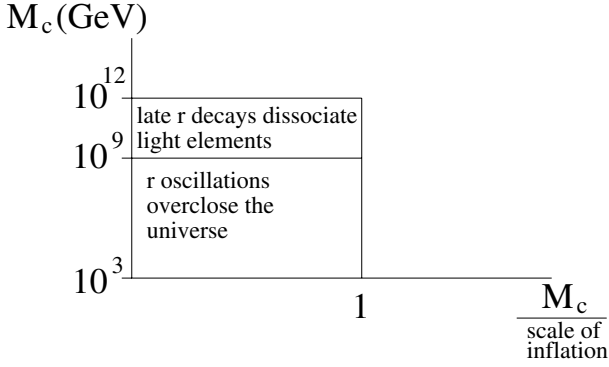
and, in  $D = 5$  it forbids the occurrence of quadratic divergences from the gauge vector boson sector. In  $D = 6$  quadratic divergences from the gauge sector can be avoided in specific models [39]. Fourth, at first sight it would seem impossible to introduce realistic Yukawa couplings, given the universality of the Higgs interactions if only minimal couplings are considered. This problem can be solved by allowing for non-local interactions induced by Wilson lines [36, 40]. Left and right-handed fermions are introduced at the opposite ends of the extra dimension (see Fig. 5). They feel only the gauge transformations that do not vanish at  $y = 0, L$ , namely those of  $SU(2) \times U(1)$  and the residual transformation in (19). An interaction term invariant under both these transformations can be written in term of a Wilson line:

$$\mathcal{L}_{Yuk} = y_{ij}^f \overline{f_{Ri}}(0) P[e^{i \int_0^L dy A_5^{\hat{a}}(y) T^{\hat{a}}}] f_{Lj}(L) + h.c. \quad (20)$$

These generic interactions should be further constrained since the couplings in  $\mathcal{L}_{Yuk}$  may reintroduce quadratic divergences for  $m_H$  at one or two-loop level.

This interesting framework has been largely developed and improved in the last year. It is based on a compactification mechanism able to provide the electroweak symmetry breaking sector starting from pure gauge degrees of freedom. It is characterized by the presence of KK gauge vector bosons of an extended group, not necessarily accompanied by KK replica for the ordinary fermions.

In a more radical proposal, the electroweak breaking itself originates directly from compactification, without the presence of explicit Higgs doublet(s) [41]. Higgsless theories of electroweak interactions in  $D=4$  are well known. One of their main disadvantages is that they become strongly interacting at a relatively low energy scale, around 1 TeV, thus hampering the calculability of precision observables. In a five-dimensional realization an ap-



**Fig. 6.** Cosmological effects of a radion mass of the order  $M_c^2/M_{Pl}$

appropriate tower of KK states can delay the violation of the unitarity bounds beyond about 10 TeV and the low-energy theory remains weakly interacting well above the four dimensional cut-off. Very recent developments show that such a possibility, at least in its present formulation, it is not compatible with the results of the precision tests [42].

**3.3 Additional remarks**

Above we have implicitly assumed that some dynamics stabilizes the radion VEV at the right scale  $M_c \equiv 1/L \approx 1$  TeV. Radion-matter interactions are controlled by the gravitational coupling  $G_N \propto 1/M_{Pl}^2$ . Other radion properties depend on the specific stabilization mechanism. In several explicit models of weak scale compactification the radion mass is approximately given by  $M_c^2/M_{Pl} \approx 10^{-3}$  eV. This induces calculable deviations from Newton’s law at distances of the order 100  $\mu\text{m}$ , even in the absence of contributions from KK graviton modes [43]. Moreover, such a light radion can dangerously modify the property of the early universe [44], if the scale of inflation is larger than the compactification scale (see Fig. 6).

There are three general problems that affect theories where the presence of ED requires a cut-off scale around 1 TeV. These theories include both the LED scenario discussed in Sect. 2 and the TeV compactification we are discussing here.

• **Conflict with EWPT**

Potentially large corrections to the electroweak observables arise when the compactification scale is close to the TeV range, from the tree-level exchange of KK gauge bosons. For  $\delta > 1$  the sum over the KK levels diverges, a regularization is needed and the related effects become sensitive to the ultraviolet completion of the theory. For  $\delta = 1$ , KK gauge boson exchange and the mixing between ordinary and KK gauge bosons typically require to push the compactification scale beyond few TeV [45]. In the LED and RS scenarios, these effects are model dependent and more difficult to estimate [46].

• **B and L approximate conservation**

Present data suggest that baryon and lepton numbers B and L are approximately conserved in nature. Stringent experimental bounds have been set on the proton lifetime. For instance [47]

$$\tau(p \rightarrow e^+\pi^0) > 4.4 \times 10^{33} \text{ yr} \quad . \quad (21)$$

Neutrino masses are extremely small compared to the other fermion masses

$$\Delta m_{atm}^2 \approx 2 \times 10^{-3} \text{ eV}^2 \quad \sum_i m_i^\nu < 1 \text{ eV} \quad . \quad (22)$$

While there are no fundamental principles requiring exact B/L conservation and indeed B/L violations are needed by baryogenesis, these experimental results imply that the amount of B/L breaking should be tiny in nature. In a low-energy (SUSY and R-parity invariant) effective theory B and L violations are described, at leading order, by dimension five operators such as

$$\int d^2\theta \frac{(H_u L)(H_u L)}{\Lambda} = \frac{v^2}{2\Lambda} \nu\nu + \dots$$

$$\int d^2\theta \frac{QQQL}{\Lambda} \quad . \quad (23)$$

(Without SUSY the leading B-violating operators have dimension six). If the cut-off  $\Lambda$  of the low-energy effective theory is very large, as the conventional solution of the hierarchy problem via four-dimensional SUSY seems to indicate, then B and L violating operators are sufficiently suppressed (we will reconsider this issue for the baryon number later on). In a theory characterized by a very low cut-off  $\Lambda$ , as the higher-dimensional theories discussed so far, additional mechanisms to deplete B/L violating operators should be invoked [9, 10].

• **Gauge coupling unification**

One of the few successful experimental indications in favor of low-energy SUSY is provided by gauge coupling unification. In the one-loop approximation the prediction for  $\alpha_3(m_Z)$  perfectly matches the experimental value [48]

$$\alpha_3(m_Z) = 0.117 \pm 0.002 \quad . \quad (24)$$

The inclusion of two-loop effects, thresholds effects and non-perturbative contributions raises the theoretical error up to  $\delta\alpha_3(m_Z) \approx 0.01$  while maintaining a substantial agreement with data. In  $D = 4$  gauge coupling unification requires three independent ingredients: logarithmic running, right content of light particles and appropriate unification conditions. If the ultraviolet cut-off, implied by the presence of extra dimensions, is much smaller than the grand unified scale, then gauge coupling unification becomes a highly non-trivial property of the theory.

It is not possible to review here all the existing proposals in order to reconcile extra dimensions having a low cut-off with B and L approximate conservation and



gauge coupling unification. Summarizing very crudely the state of the art we can say that these properties are quite natural within the ultraviolet desert of a (SUSY) four-dimensional low-energy theory, especially if such a theory is complemented by a grand unified picture where the particle classification is clarified and the required unification condition is automatic. On the other hand, B and L approximate conservation and gauge coupling unification are possible [9,10,49], but far from generic features if an infrared desert exist:  $V_\delta \gg M_D^{-\delta}$ . In the absence of a desert, as in the case of compactifications at the TeV scale, gauge coupling unification is lost <sup>3</sup>. In the Randall-Sundrum setup the TeV scale is not the highest accessible energy scale and this leaves open the possibility of achieving gauge coupling unification in a way close to the conventional four-dimensional picture [52].

## 4 Grand unification and extra dimensions

The idea of grand unification is extremely appealing. It automatically provides the unification conditions for a successful gauge coupling unification. It sheds light on the otherwise mysterious classification of matter fields, giving a simple explanation for electric charge quantization, gauge anomaly cancellation and suggesting relations among fermion masses. It provides the ingredients for baryogenesis. However, in its conventional four-dimensional formulation, grand unification is affected by two major problems.

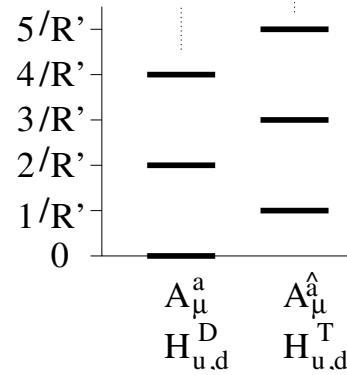
The first is the doublet-triplet splitting problem. In any grand unified theory (GUT) the electroweak doublets  $H_{u,d}^D$ , needed for the electroweak symmetry breaking, sit in the same representation of the grand unified group together with color triplets  $H_{u,d}^T$ . Doublets are required to be at the electroweak scale, while triplets should be at or above the unification scale, to avoid fast proton decay and not to spoil gauge coupling unification. Such a huge splitting is fine-tuned in minimal models, or realized naturally at the price of baroque Higgs structures in non-minimal ones [53]. Moreover, even when implemented at the classical level, it can be upset by radiative corrections after SUSY breaking and by the presence of non-renormalizable operators induced by physics at the  $M_{Pl}$  scale [54].

The second problem is the conflict between the proton decay rates, dominated by dimension five operators in minimal SUSY GUTs, and experimental data. For instance minimal SUSY SU(5) predicts [55]:

$$\tau(p \rightarrow K^+ \bar{\nu}) \approx 10^{32} \left( \frac{2 \tan \beta}{1 + \tan^2 \beta} \right)^2 \left( \frac{m_T(\text{GeV})}{10^{17}} \right)^2 \text{ yr} . \quad (25)$$

This prediction is affected by several theoretical uncertainties (hadronic matrix elements, masses of supersymmetric particles, additional physical phase parameters, wrong

<sup>3</sup> The possibility that power-law running [50] enforces gauge coupling unification at a scale close to the TeV depends on assumptions about the unknown ultraviolet completion of the theory [51].



**Fig. 7.** Mass spectrum in the gauge boson and Higgs sectors of a five-dimensional SU(5) GUT, compactified on  $S/(Z_2 \times Z_2)$

mass relations between quarks and leptons of first and second generations) but, even by exploiting such uncertainties to stretch the theoretical prediction up to its upper limit, the conflict with the present experimental bound

$$\tau(p \rightarrow K^+ \bar{\nu}) > 1.9 \times 10^{33} \text{ yr} \quad (90\% \text{ CL}) \quad , \quad (26)$$

is unavoidable (in (25) such uncertainties have already been optimized to minimize the rate).

Remarkably, both these problems can be largely alleviated if the grand unified symmetry is broken by compactification of a (tiny) extra dimension on a orbifold [56]. The gauge symmetry breaking mechanism is exactly the one we have already described when discussing gauge-higgs unification. For instance, in the case of SU(5), the gauge vector bosons  $A_\mu$  can be assembled in a  $5 \times 5$  hermitian traceless matrix as follows:

$$A_\mu = \left( \begin{array}{c|c} A_\mu^a & A_\mu^{\hat{a}} \\ \hline A_\mu^{\hat{a}} & A_\mu^a \end{array} \right) , \quad (27)$$

where the diagonal blocks are  $3 \times 3$  and  $2 \times 2$  matrices. Then  $A_\mu^a$  ( $a = 1, \dots, 12$ ) are the gauge vector bosons of the SM while  $A_\mu^{\hat{a}}$  ( $\hat{a} = 13, \dots, 24$ ) are associated to the generators that are in SU(5) and not in the SM. By working in five dimensions, the zero modes of  $A_\mu^{\hat{a}}$  can be eliminated if we require that these fields are odd under the parity symmetry  $Z_2'$ :  $y' \rightarrow -y'$ ,  $y' \equiv y - \pi R'/2$ . The resulting spectrum is displayed in Fig. 7. If we consider a tiny radius of the fifth dimension,  $1/R' \approx M_{GUT}$  ( $M_{GUT} = 2.4 \times 10^{16}$  GeV being the four-dimensional grand unified scale), then from the viewpoint of the four-dimensional observer, having access to energies much smaller than  $M_{GUT}$ , SU(5) appears to be broken down to  $SU(3) \times SU(2) \times U(1)$ . Moreover, if the Higgs multiplets  $H_{u,d}$  containing the Higgs doublets  $H_{u,d}^D$  live in the bulk, their  $Z_2'$  parity assignment is fixed from the gauge sector (up to a twofold ambiguity) and the desired DT splitting can be automatically achieved by compactification [56], as illustrated in Fig. 7. In the SUSY version of this model another parity  $Z_2$ , acting as  $y \rightarrow -y$ , removes all the additional states required

by SUSY in D=5 and the massless modes are just in  $A_\mu^a$  and  $H_{u,d}^D$ . The double identification implied by  $Z_2 \times Z_2'$  reduces the circle  $S$  down to the interval  $(0, \pi R'/2)$ . The point  $y = \pi R'/2$  is special, since all the gauge vector bosons  $A_\mu^a$  and the parameters of the corresponding gauge transformations vanish there. Therefore in  $y = \pi R'/2$  the effective gauge symmetry is only  $SU(3) \times SU(2) \times U(1)$ , not the full  $SU(5)$ .

To complete the solution of the DT splitting problem and to keep under control proton decay, the mass term  $H_u H_d$  allowed by both  $SU(5)$  gauge symmetry and  $N = 1$  five-dimensional supersymmetry [57], should be adequately suppressed. This can be done by assuming a  $U(1)_R$  symmetry of Peccei-Quinn type, broken down to the usual  $R$ -parity only by small supersymmetry breaking soft terms [58]. This symmetry keeps the Higgs doublets light and, when extended to the matter sector to include  $R$ -parity, forbids dimension five operators leading to proton decay (see (23)) and eliminates all dangerous B/L violating operators of dimension four. Alternatively we can assign appropriate parities to matter fields, to suppress or even to completely forbid proton decay, at the price of explicitly loosing the  $SU(5)$  gauge symmetry in the matter sector [59].

Gauge coupling unification is preserved [58, 60]. There are several contributions to the running of gauge coupling constant beyond the leading order. As in the conventional four-dimensional analysis we have two-loop corrections and contributions coming from the light thresholds, the masses of the superpartners of the ordinary particles. These corrections raise the leading order prediction of  $\alpha_3(m_Z)$ :

$$\alpha_3(m_Z) = \alpha_3^{LO}(m_Z) + \delta^{(2)}\alpha_3 + \delta^{(light)}\alpha_3 \approx 0.130 \quad . \quad (28)$$

An additional correction, specific of the present framework, comes from possible gauge kinetic terms localized at the brane at  $y = \pi R'/2$ , where the effective gauge symmetry is only  $SU(3) \times SU(2) \times U(1)$ , not the full  $SU(5)$ :

$$\sum_{i=1,2,3} \frac{1}{g_{bi}^2} \int d^4x dy \delta(y - \frac{\pi R'}{2}) F_{\mu\nu_i} F_i^{\mu\nu} \quad . \quad (29)$$

Even if these terms are absent at the classical level, they are expected to arise from divergent radiative corrections [61]. Therefore a consistent description of the theory requires their presence, with  $g_{bi}$  as free parameters. By taking into account both the five-dimensional gauge kinetic term and the contributions in (29), the gauge coupling constants  $g_i^2$  at the cut-off scale  $\Lambda$  are given by

$$\frac{1}{g_i^2} = \frac{2\pi R'}{g_5^2} + \frac{1}{g_{bi}^2} \quad . \quad (30)$$

If the  $SU(5)$  breaking terms  $1/g_{bi}^2$  were similar in size to the symmetric one, we would loose any predictability. A predictive framework can be recovered by assuming that at the scale  $\Lambda$  the theory is strongly coupled. In this case  $g_5^2 \approx 16\pi^3/\Lambda$ ,  $g_{bi}^2 \approx 16\pi^2$  and from (30) we estimate

$$\frac{1}{g_i^2} \approx \frac{\Lambda R'}{8\pi^2} + O\left(\frac{1}{16\pi^2}\right) \quad . \quad (31)$$

The  $SU(5)$  symmetric contribution dominates over the brane contributions and predicts gauge couplings of order one, provided  $\Lambda R' = O(100)$ . Such a gap between the compactification scale  $M_c \equiv 1/R'$  and the cut-off scale  $\Lambda$  can in turn independently affect gauge coupling unification through heavy threshold corrections. At leading order these corrections, coming from the particle mass spectrum around the scale  $M_c$ , are given by [58]:

$$\delta^{(heavy)}\alpha_3 = -\frac{3}{7\pi}(\alpha_3^{LO})^2 \log\left(\frac{\Lambda}{M_c}\right) \quad . \quad (32)$$

It is quite remarkable that  $\Lambda R' = O(100)$  is precisely what needed in order to compensate the corrections in (28) and bring back  $\alpha_3(m_Z)$  to the experimental value (24). By considering the whole set of renormalization group equations one also finds the preferred values

$$\Lambda \approx 10^{17} \text{ GeV} \quad M_c \approx 10^{15} \text{ GeV} \quad . \quad (33)$$

The compactification scale  $M_c \approx 10^{15} \text{ GeV}$  is rather smaller than  $M_{GUT}$  and this greatly affects the estimate of the proton lifetime. The presence of non-universal brane kinetic terms, as given by the strong coupling estimate in (31), suggests that the theoretical error on the prediction of  $\alpha_3(m_Z)$  is similar to the one affecting the four-dimensional  $SU(5)$  analysis.

The fifth dimension has also an interest impact on the description of the flavour sector. Each matter field can be introduced either as a bulk field, depending on all the five-dimensional coordinates, or as a brane field, located at  $y = 0$  or in  $y = \pi R'/2$ . Matter fields living in  $y = \pi R'/2$ , can in principle be assigned to representations of  $SU(3) \times SU(2) \times U(1)$  that do not form complete multiplets under  $SU(5)$ . For instance we could replace the Higgs multiplets, so far regarded as bulk fields, with brane four-dimensional fields localized in  $y = \pi R'/2$ . In this case we could avoid the inclusion of the colour triplet components, limiting ourselves to the  $SU(2)$  doublets alone. This possibility would provide a radical solution to the DT splitting problem [62], which cannot be contemplated in the four-dimensional construction.

To maintain the power of  $SU(5)$  in particle classification, it is preferable to introduce fermions and their supersymmetric partners as bulk fields or as brane fields localized at  $y = 0$ , where the full  $SU(5)$  symmetry is effective. Zero modes from fermion bulk fields differs from brane fermions in two respects. First, each bulk field experiences the same splitting that characterizes the gauge and the Higgs multiplets (see Fig. 7). Therefore to produce the zero modes of a complete  $SU(5)$  representation two identical bulk multiplets, with opposite  $Z_2'$  parities are needed. Second the zero modes of bulk fields have a  $y$ -constant wave function carrying the characteristic suppression  $\epsilon \equiv 1/\sqrt{\Lambda R}$  (see (8)), which, as we have seen before, is of the same order of the Cabibbo angle. As a consequence, Yukawa couplings between zero modes arising from bulk fields are depleted with respect to those between brane fields, the relative suppression factor being  $\epsilon^2$ ,  $\epsilon$  and 1, respectively, for bulk-bulk, bulk-brane, brane-

brane interactions. Moreover only brane-brane Yukawa interactions can lead to the SU(5) mass relation  $m_e = m_d$ , since the doubling of SU(5) representations for bulk matter fields lead to SU(5)-unconstrained couplings between the zero modes. All this suggest to localize the third generation on the  $y = 0$  brane, while choosing bulk fields for at least a part of the matter in the first and second generation. In this way the successful relation  $m_b = m_\tau$  of minimal SU(5) is maintained, while the unwanted analogues relations for the first two generations are lost.

The most relevant signature of GUTs is represented by proton decay. In the five-dimensional SU(5) model under discussion, this process is dominated by the exchange of the gauge vector bosons  $A_\mu^{\hat{a}}$  [58, 63]. In minimal SUSY SU(5) such a contribution is controlled by the unification scale  $M_{GUT} \approx 10^{16}$  GeV and, by itself, would give rise to a proton life of the order  $10^{36}$  yr, too long to be observed at present and foreseen facilities. On the contrary, in the five-dimensional SU(5) realization, the masses of the lightest gauge vector bosons  $A_\mu^{\hat{a}}$  are at the compactification scale  $M_c \approx 10^{15}$  GeV, which means an enhancement of four order of magnitudes in the proton decay rate. Such a huge enhancement is in part balanced by suppression factors coming either from the mixing angles needed to relate the third generation living at  $y = 0$  to the lightest generations, or from non-minimal brane couplings between  $A_\mu^{\hat{a}}$  and light bulk fermions. These suppression factors are also the main source of the large uncertainty in the estimate of the proton lifetime. On the other hand, all the uncertainties coming from the supersymmetry breaking sector of the theory, which affect  $p$ -decay dominated by the triplet higgsino exchange, are absent. The proton lifetime is expected to be close to  $10^{34}$  yr and the main decay channels are  $e^+\pi^0$ ,  $\mu^+\pi^0$ ,  $e^+K^0$ ,  $\mu^+K^0$ ,  $\bar{\nu}\pi^+$ ,  $\bar{\nu}K^+$ .

This framework has also been extended to larger grand unified groups like SO(10) and  $E_6$  [64]. The gauge symmetry breaking of the GUT symmetry down to the SM one is accomplished partly by the compactification mechanism that, in its simplest realization, does not lower the rank of the group and requires more than one ED and partly by a conventional Higgs mechanism.

## 5 Flavour problem

A realistic description of fermion masses in a four dimensional framework typically requires either a large number of parameters or a high degree of complexity and we are probably unable to select the best model among the very many existing ones. Moreover, in four dimensions we have little hopes to understand why there are exactly three generations. These difficulties might indicate that at the energy scale characterizing flavour physics a four-dimensional description breaks down. This happens in superstring theories. In the ten-dimensional heterotic string six dimensions can be compactified on a Calabi-Yau manifold [65] or on orbifolds [66] and the flavour properties are strictly related to the features of the compact space. In Calabi-Yau compactifications the number

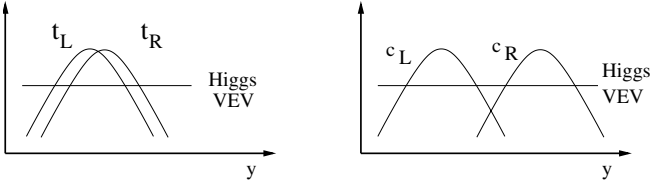
of chiral generations is proportional to the Euler characteristics of the manifold. In orbifold compactifications, matter in the twisted sector is localized around the orbifold fixed points and their Yukawa couplings, arising from world-sheet instantons, have a natural geometrical interpretation [67]. Recently string realizations where the light matter fields of the SM arise from intersecting branes have been proposed. Also in this context the flavour dynamics is controlled by topological properties of the geometrical construction [68], having no counterpart in four-dimensional field theories.

It has soon been realized that also in a field theoretical description the existence of extra dimensions could have important consequences for the flavour problem. For instance in orbifold compactifications light four-dimensional fermions may be either localized at the orbifold fixed points or they may arise as zero modes of higher-dimensional spinors, with a wave function suppressed by the square root of the volume of the compact space (see (8)). This led to several interesting proposals. For instance, as already discussed in Sect. 2.1.2, we can describe neutrino masses by allowing right-handed sterile neutrinos to live in the bulk of a large fifth dimension [8]. We have also seen that in five-dimensional grand unified theories the heaviness of the third generation can be explained by localizing the corresponding fields on a fixed point, whereas the relative lightness of the first two generations as well as the breaking of the unwanted mass relations can be obtained by using bulk fields [58, 69].

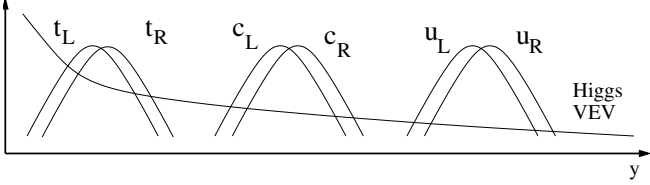
Even more interesting is the case when a higher dimensional spinor interacts with a non-trivial background of solitonic type. It has been known for a long time that this provides a mechanism to obtain massless four-dimensional chiral fermions [70, 71]. For instance, the four-dimensional zero modes of a five-dimensional fermion ( $\psi_L, \psi_R$ ) interacting with a real scalar background  $\varphi(x_5)$  are formally given by

$$\psi_{L,R}(x, x_5) \propto e^{\pm g \int_{x_5^0}^{x_5} du \varphi(u)} \psi_{L,R}(x) \quad . \quad (34)$$

If  $\varphi(x_5)$  is a soliton, with  $\varphi(\pm\infty) = \pm\varphi_\infty$  ( $\varphi_\infty > 0$ ) only one of the two solutions in (34) is normalizable:  $\psi_L$  ( $\psi_R$ ) if  $g > 0$  ( $g < 0$ ). Moreover, since the wave function is localized around the core  $x_5 = x_5^0$  of the topological defect, where  $\varphi(x_5^0) = 0$ , such a mechanism can play a relevant role in explaining the observed hierarchy in the fermion spectrum [10]. Mass terms arise dynamically from the overlap among fermion and Higgs wave functions (see Figs. 8 and 9). Typically, there is an exponential mapping between the parameters of the higher-dimensional theory and the four-dimensional masses and mixing angles, so that even with parameters of order one large hierarchies are created [72]. In orbifold compactifications, solitons are simulated by scalar fields with a non-trivial parity assignment that forbids constant non-vanishing VEVs. Also in this case the zero modes of the Dirac operator in such a background can be chiral and localized in specific regions of the compact space.



**Fig. 8.** A different relative localization of left and right-handed wave functions for *top* and *charm* quarks produces different overlaps with a constant Higgs VEV

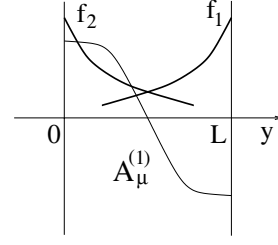


**Fig. 9.** An equal relative localization of left and right-handed wave functions for up-type quarks produces different overlaps with a non-uniform Higgs VEV

A quite interesting possibility arising in models of this sort, is that several zero modes can originate from a single higher-dimensional spinor [70,71], thus providing an elegant mechanism for understanding the fermion replica. For instance, in the model studied in [73] there is a vortex solution that arises in the presence of two infinite extra dimensions. It is possible to choose the vortex background in such a way that the number of chiral zero modes of the four-dimensional Dirac operator is three. Each single six-dimensional spinor gives rise to three massless four-dimensional modes with the same quantum numbers. Recently this model has been extended to the case of compact extra dimensions [74].

In orbifold compactifications similar results can be obtained. Matter can be described by vector-like  $D$ -dimensional fermions with the gauge quantum numbers of one SM generation. As a result, the model has neither bulk nor localized gauge anomalies. The different generations arise as zero modes of the four-dimensional Dirac operator by eliminating the unwanted chiralities of the  $D$ -dimensional spinors through an orbifold projection. By consistency,  $D$ -dimensional fermion masses are required to transform non-trivially under the discrete symmetry defining the orbifold and, as a consequence, the independent zero modes are localized in different regions of the extra space. If the Higgs VEV is not constant in the extra space, but concentrated around some particular point, fermion masses will acquire the desired hierarchy (see Fig. 9). A toy model successfully implementing this program in the case of two fermion generations has been recently build [75], by working with two extra dimensions compactified on a orbifold  $T^2/Z_2$ . In this model a non-trivial flavour mixing is related to a soft breaking of the six dimensional parity symmetry. In particular, the empirical relation  $\theta_C \approx \sqrt{m_d/m_s}$  can be easily accommodated.

The possibility of testing experimentally this idea is strictly related to the typical size of the extra dimension involved. When fermions of different generations have



**Fig. 10.** The non-constant wave function of the KK gauge vector boson  $A_\mu^{(1)}$  can give rise to different coupling to first ( $f_1$ ) and second ( $f_2$ ) fermion generations

wave functions with different profiles along the extra dimensions, new sources of flavour violation appear [76]. Higher KK gauge boson excitations have non-constant wave functions and this gives rise to non-universal interactions with ordinary fermions in four dimensions (see Fig. 10). Exchange of KK gauge bosons produce four-fermion interactions, which, after rotation from flavour to mass eigenstate basis, mediate flavour-changing neutral currents. The current limits on FCNC can probe compactification scales up to about 100 TeV.

## 6 Cosmological constant problem

To a good approximation, our four-dimensional space-time possesses an approximate Poincaré symmetry and, by the law of general relativity, this is attributed to a tiny vacuum energy density  $\Lambda_{CC}$  of our universe. There are two puzzling aspects of such an interpretation. First, from the knowledge of the mass scales implied in fundamental interactions we would estimate typical values of  $\Lambda_{CC}$  that are many order of magnitudes bigger than the “observed” one. Second, we do not understand why the vacuum energy density is of the same order of magnitude of the matter energy density today. We conclude this review with a qualitative comment about the relevance of extra dimensions to the first aspect of the problem.

In four space-time dimensions it is not possible to achieve a natural cancellation of the cosmological constant (which would represent already a good starting point to understand its smallness) [77]. Very roughly, in four dimensions the requirement of general covariance imply the existence of a massless graviton, universally coupled to all sources. Therefore, independently from the size of the source, the gravitational coupling is always given by the Newton constant  $G_N = 1/(8\pi M_{Pl}^2)$ . The laws of general relativity demand that the vacuum energy of the universe curves our space-time by the amount:

$$H^2 \propto G_N \Lambda_{CC} \propto \frac{\Lambda_{CC}}{M_{Pl}^2} \quad (35)$$

Notice that the present measurements are only sensitive to the left-hand side, and the tiny value  $\Lambda_{CC} \approx (10^{-3}\text{eV})^4$  is a consequence of the universal graviton coupling, which in four dimensions cannot be questioned.

In more than four space-time dimensions this conclusion is not inescapable since, under certain conditions,  $\Lambda_{CC}$  can curve the extra space leaving our space-time essentially flat [78]. For instance, this is what happens, at the price of a fine-tuning, in the Randall-Sundrum model described in Sect. 2.2. If these conditions could be naturally enforced, this would allow to reconcile a large vacuum energy density  $\Lambda_{CC}$  with the observed smallness of the four-dimensional space-time curvature  $H$ . The four-dimensional observer would interpret the absence of space-time curvature as a modification of gravity at very large distances, with a substantial reduction of the gravitational coupling to the vacuum energy density. Such a mechanism might take place in the presence of an effective gravitational coupling  $G_N(\lambda)$  depending on the wavelength of the source. For instance, for wavelengths smaller than the present Hubble distance,  $G_N(\lambda)$  could coincide with the Newton constant

$$G_N(\lambda) = \frac{1}{8\pi M_{Pl}^2} \quad \lambda \leq H_0^{-1} \approx 10^{28} \text{ cm} \quad , \quad (36)$$

not to induce deviations from the standard cosmology. For larger wavelengths, the gravitational coupling could be much smaller:

$$G_N(\lambda) \ll \frac{1}{8\pi M_{Pl}^2} \quad \lambda > H_0^{-1} \approx 10^{28} \text{ cm} \quad . \quad (37)$$

Since the vacuum energy represents the source with the largest wavelength  $\lambda \gg H_0^{-1}$ , a large  $\Lambda_{CC}$  could induce a relatively small four-dimensional space-time curvature, compatible with the present observations.

Until recently there were no explicit examples of consistent theories where the behaviour of gravity is modified at large distances. A substantial progress is represented by models where the SM fields are localized on a brane in infinite volume ED [79]. In these models gravity along the brane changes from a four-dimensional regime at small distances to a higher-dimensional regime at very large distances and this gives rise to an effective gravitational coupling  $G_N(\lambda)$  with the kind of dependence described above [80].

The existence of a large hierarchy between two mass scales is not avoided in these models. In particular, to make the cross-over distance sufficiently large, the scale of gravity in the higher-dimensional theory should be quite small, of the order of  $10^{-3}$  eV. This hierarchy can however be made technically stable. An interesting feature is represented by expected modifications of the Newton's law at distances below 1 mm, an aspect that is also common to other approaches to the cosmological constant problem [81], where point-like gravity breaks down around the 100  $\mu\text{m}$  scale. The physical implications of this class of models are presently under investigation and it is not clear if the difficulties related to the effective low-energy description [82] can be overcome by searching for an embedding in the context of a fundamental theory such as string theory [83]. It is nevertheless interesting to have concrete examples where non-local modifications of gravity, consistent with the equivalent principle, can be analyzed [84].

*Acknowledgements.* I am grateful to Guido Altarelli, Carla Biggio, Andrea Brignole, Antonio Masiero, Isabella Masina, Manuel Perez-Victoria, Riccardo Rattazzi, Tony Riotto and Fabio Zwirner for useful discussions while preparing this talk.

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